Integrals: A Brief Introduction

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Calculus is fundamentally about finding and using two quantities: the *derivative* and the *integral*. When using the methods of mathematicians who invented calculus, one will differentiate equations to find derivatives and integrate to find integrals. Calculus is also a branch of mathematics that deals mostly with curves, which are functions that change how they change. (Whereas a straight line behaves in the same way no matter its distance from the origin, curves change how they change as they get away from the origin-- sure this is a simplification, but it does illustrate the point.)

The derivative is defined as the slope of a line tangent to a point along a graph. We understand this to be a measure of the rate at which the graph is changing along its path. For simple situations, we can find the derivative of of a graph at a point using the general power rule:

If, $f(x)=x^r$, then we find its derivative $f'(x)=r\,x^{(r-1)}$

Finding the indefinite integral is sometimes called finding the antiderivative. If one is given the derivative of an equation, one can do the anti of taking the derivative to find the integral:

If $f'(x)=x^r$, then we find its integral, $\int f(x)\,dx$: $\int x^r\,dx=\frac{1}{r+1}x^{r+1}$

Let's consider an example: If a rocket is launched vertically and its velocity at time *t* is given by the equation $v(t)=20\mathrm{t}+50$ (m/s), then we can make predictions about its height at various times into its flight.

(Before proceeding, understand the motion we are describing: at *t* = 0, the rocket had a velocity of 50 m/s-- don't worry about the difficulty of doing that for now-- and it must be accelerating, as we can see that when *t* = 1, *v* is going to equal 70; and at *t* = 2, *v* = 90.)

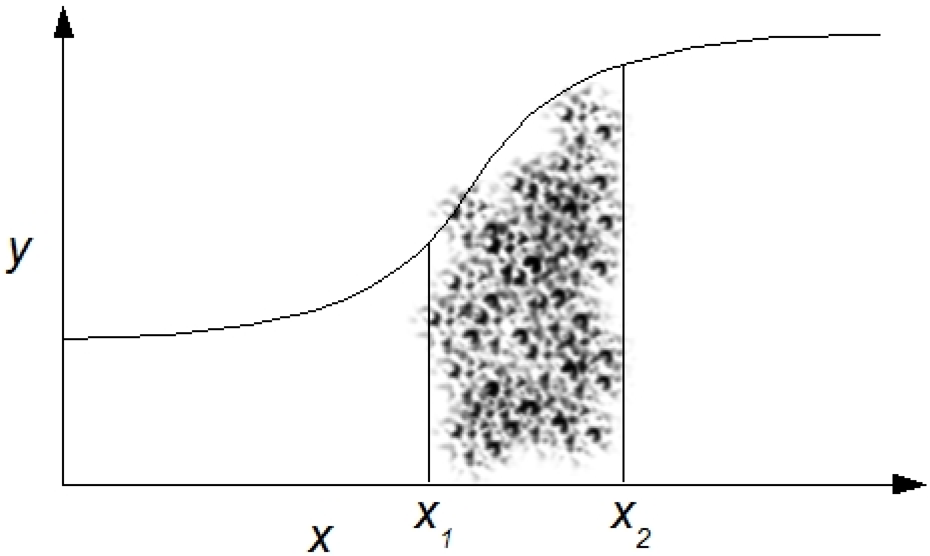
If we integrate (using the anti of the general power rule) the equation *v(t)*, we find an equation that will predict for us, the height of the rocket (because we are familiar with physics of rockets that are accelerating, we can expect to find a quadratic equation):

$$s(t)=\int v(t)\,dt=\int (20\mathrm{t}+50)\,dt$$
$$s(t)=(20)(1/2)t^2+50t$$
$$s(t)=10t^2+50t$$

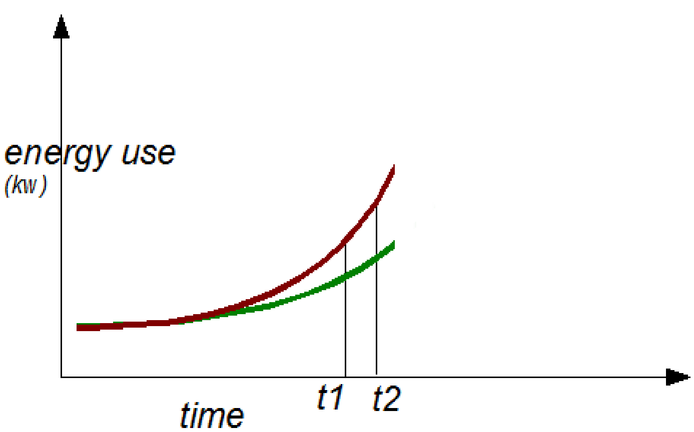
Frequently, one is interested in the definite integral, the integral between two values along a graph. This is called finding a definite integral:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ (where *F(b)* is the antiderivative at b... and so for *F(a)* also)

The define integral is useful for finding the area under a curve:



If we find the integral at x_2 and subtract the integral at x_1, then we can find the area between the graph and the *x* axis-- a colored the area in the picture (pretend I can color inside the lines!).



This is sometime useful for finding differences... let's say the energy consumption of a city is given by the red line, then and energy conservation scheme is instituted, and the new energy consumption is shown by the green graph. If we find the difference between the integrals of the red graph and the green graph between any dates, then we can estimate how much energy was saved by the scheme.